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2006 J. Phys.: Condens. Matter 18 S2083

(http://iopscience.iop.org/0953-8984/18/33/S24)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **18** (2006) S2083–S2093 doi:10.1088/0953-8984/18/33/S24

Radiation of relativistic particles for quasiperiodic motion in a transparent medium

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Received 9 March 2006
Published 4 August 2006
Online at stacks.iop.org/JPhysCM/18/S2083

Abstract
The radiation of relativistic charged particles for quasiperiodic motion in a transparent medium is considered. For motion of the general kind the differential probability of the process is obtained. For planar motion the spectral intensity of the radiation is found. Different cases of radiation in medium-filled undulators are studied. In particular, the influence of Cherenkov radiation on the undulator radiation is discussed.

1. Introduction

At the present time such power sources of x-rays as undulators [1] are widely used in various fields of science. In a number of papers [2–9], with the aim of increase of the energy of emitted photons, the crystal undulator has been considered. The recent paper [10] contains a rather complete list of references related to various problems of crystal undulators[3].

One of the proposed constructions [12, 13] was created and tested in a positron beam. Preliminary results of the experiment [14] give an indication on the observation of undulator radiation. Calculations of the expected intensity for this experiment were based on the theory [15] of radiation for quasiperiodic motion in vacuum. This theory allows one to perform the calculation of radiation spectra for motion of the general type and different parameters of the undulator. In calculations with our experimental conditions we use the theory [15] in the framework of classical electrodynamics.

However, in the recent papers [16, 17] the possibility of an appreciable influence of the medium polarization on the spectral intensity in crystal undulators was shown. In [16] the process was considered only in the dipole approximation. In [17] this process was studied for a specific construction of the undulator and hence for specific trajectories of particles. In both cases the radiation of the first harmonic was considered.

In this paper we want to extend the theory [15] to the case of a transparent medium. We will study the pointed out process in the general case of a dielectric function $\varepsilon(\omega)$ (where ω

[3] We, following the authors of [3, 11] (among the first publications in this field), cannot agree with some statements in the article [10] which touch upon some historical aspects and assessments of the contributions of different authors to this issue.

is the frequency or energy of the emitted photon) which may be larger or smaller than 1. The phenomena arising in different cases will be briefly discussed. In this paper we employ units such that $\hbar = c = 1$.

Note that a large number of problems of radiation for charged particles moving in various media were considered in [18]. Here, investigations of the radiation in medium-filled undulators are also presented. However, these results concern mainly the total radiation intensity. We also point out the paper [19] where the radiation for quasiperiodic motion was studied for a wide range of undulator parameters.

2. Radiation energy losses of particles

The well-known formula [20, 21] for the radiation energy losses of a moving particle takes into account the dielectric function of the medium. The analogous formula in [15] differs from the above pointed one and was obtained for the vacuum. With the aim of extending the theory [15] (in the framework of classical electrodynamics) to the case of a transparent medium we should find its corresponding representation.

The Fourier transform of the vector potential for the electromagnetic field of a charged particle moving in an isotropic transparent medium has the following form [21]:

$$\mathbf{A}(\omega, r) = e\frac{\exp[\mathrm{i}kr]}{r}\int \mathbf{v}(t)\exp\{\mathrm{i}[\omega t - \mathbf{k}\mathbf{r_0}(t)]\}\,\mathrm{d}t, \tag{1}$$

where $\mathbf{k} = \sqrt{\varepsilon}\omega\mathbf{n}$, $\varepsilon(\omega)$ is the dielectric function (ε is a real positive value), $\mathbf{n}$ is the unit vector in the direction of the photon motion, $\mathbf{v}$, $\mathbf{r}_0$ are the particle velocity and its radius vector, r is the distance from the point where the particle (with the charge e) is located at the moment of time t. This relation is valid for large r.

Using this equation we find (analogously to [15]) the magnetic and electric ($\mathbf{E}$) fields. The radiated energy $\mathrm{d}\mathcal{E}(\mathbf{n}, \omega)$ in an elementary solid angle $\mathrm{d}\Omega$ and a frequency range ω, $\omega + \mathrm{d}\omega$ for the whole time of the process is [22]

$$\mathrm{d}\mathcal{E}(\mathbf{n}, \omega) = \sqrt{\varepsilon}|\mathbf{E}(\omega)|^2(\mathrm{d}\omega/4\pi^2)\,\mathrm{d}\Omega r^2. \tag{2}$$

Finally, we obtain

$$\mathrm{d}\mathcal{E}(\mathbf{n}, \omega) = e^2\sqrt{\varepsilon}\int\int[(\mathbf{v}(t_1)\mathbf{v}(t_2) - 1/\varepsilon]$$
$$\times \exp\{\mathrm{i}[\omega(t_1 - t_2) - \mathbf{k}[\mathbf{r_0}(t_1) - \mathbf{r_0}(t_2)]]\}\frac{\omega^2\,\mathrm{d}\omega\,\mathrm{d}\Omega}{(2\pi)^2}\,\mathrm{d}t_1\,\mathrm{d}t_2. \tag{3}$$

This equation describes the differential radiation energy losses of the relativistic particle moving in a transparent medium. At $\sqrt{\varepsilon} = 1$ equation (3) is the same as in [15].

The relation obtained here contains the peculiarities of radiation processes in a medium. Let us calculate for demonstration purposes the radiation of the relativistic charged particle moving in a transparent medium with a constant velocity (Cherenkov radiation). It is easy to take the integrals over t_1 and t_2:

$$\mathrm{d}\mathcal{E}(\mathbf{n}, \omega) = e^2\sqrt{\varepsilon}(v^2 - 1/\varepsilon)T_{\mathrm{m}}\delta(\omega - \sqrt{\varepsilon}\omega v\cos\theta)\frac{\omega^2\,\mathrm{d}\omega\,\mathrm{d}\Omega}{2\pi}, \tag{4}$$

where T_{m} is the time of particle motion and θ is the polar angle, which is determined by the pair of vectors $\mathbf{v}$ and $\mathbf{n}$.

From here, we get the intensity of radiation per unit time

$$\frac{\mathrm{d}\mathcal{E}(\omega)}{T_{\mathrm{m}}} = e^2v(1 - 1/(\varepsilon v^2))\vartheta(1 - 1/(\varepsilon v^2))\omega\,\mathrm{d}\omega, \tag{5}$$

where $\vartheta(x) = 1$ at $x > 0$ and $\vartheta(x) = 0$ at $x < 0$.

3. Intensity of radiation for quasiperiodic motion

Let us suggest that the particle performs a quasiperiodic motion with the period equal to T.

The periodicity of motion allows us to transform the integral in equation (3) into a Fourier series. In accordance with [15] (conserving the notation of the variables) we can write

$$\int_{-\infty}^{\infty} v^{\mu}(t)\mathrm{e}^{\mathrm{i}kx(t)}\,\mathrm{d}t = v^{T_\mu}\sum_{m=-\infty}^{\infty}\mathrm{e}^{\mathrm{i}m\varphi_0} = 2\pi v^{T_\mu}\sum_n \delta(\varphi_0 - 2\pi n), \tag{6}$$

where

$$kx(t) = \omega t - \sqrt{\varepsilon}\omega\mathbf{n}\mathbf{r}_0(t), \qquad \varphi_0 = \omega T(1-\sqrt{\varepsilon}\mathbf{n}\mathbf{V}), \qquad v^{T_\mu} = \int_0^T v^{\mu}(t)\mathrm{e}^{\mathrm{i}kx(t)}\,\mathrm{d}t. \tag{7}$$

Here $v^{\mu} = (1, \mathbf{v})$, $(\mu = 0\text{–}3)$ is the 4-vector of the particle velocity and $\mathbf{V} = (1/T)\int_0^{\mathrm{T}} \mathbf{v}(t)\,\mathrm{d}t$ is the mean particle velocity. One can find the mean velocity V and longitudinal velocity $v_{\parallel}$:

$$V \approx 1 - (1+\overline{v_{\perp}^2}\gamma^2)/(2\gamma^2), \qquad v_{\parallel} \approx 1 - (1+v_{\perp}^2\gamma^2)/(2\gamma^2), \tag{8}$$

where $\overline{v_{\perp}^2}$ is the mean square of the transverse velocity $v_{\perp}$. Substituting equation (6) into (3) and calculating the intensity of radiation (per unit of time) one can get

$$\mathrm{d}I = e^2\sqrt{\varepsilon}\frac{\omega^2\,\mathrm{d}\omega\,\mathrm{d}\Omega}{4\pi T}\sum_{n=-\infty}^{\infty}\delta(\varphi_0 - 2\pi n)\{2(|\mathbf{v}^{\mathrm{T}}|^2 - |v^{T_0}|^2/\varepsilon)\}. \tag{9}$$

One can see that in the general case the number n may be positive as well as negative (see, for an explanation, for example [18, 19]).

Below we will find the relativistic relations with an accuracy up to γ^{-2} terms. Besides this, we will obtain the intensity of radiation for any positive value of $\sqrt{\varepsilon} = 1 + (\sqrt{\varepsilon} - 1) = 1 + \chi$. Now we find the following relations:

$$f(t) = \omega t - \sqrt{\varepsilon}\mathbf{n}\mathbf{r}_{\mathbf{0}}\omega = -\chi\omega t + \sqrt{\varepsilon}\omega\left[\frac{\theta^2 t}{2} + \frac{t}{2\gamma^2} + \int_0^t v_{\perp}^2\,\mathrm{d}t - \mathbf{n}_{\perp}\mathbf{x}_{\perp}\right], \tag{10}$$

$$\varphi_0/T = \omega\left(-\chi + \sqrt{\varepsilon}\frac{(1+\overline{v_{\perp}^2}\gamma^2)}{2\gamma^2} + \frac{\sqrt{\varepsilon}\theta^2}{2}\right) = n\omega_0. \tag{11}$$

Then we can find the spectral angular distribution of the radiation

$$\mathrm{d}I = e^2\sqrt{\varepsilon}\frac{\omega^2\,\mathrm{d}\omega\,\mathrm{d}\Omega}{(2\pi)^2}\frac{1}{\omega_0\gamma^2}\sum_{n=-\infty}^{\infty}\delta(\varphi_0 - 2\pi n) \times\left[\left\{\frac{(\varepsilon-1)}{\varepsilon}\gamma^2 - 1\right\}|I_0|^2 + \gamma^2(|\mathbf{I}_{\perp}|^2 - Re I_0^* I_{\parallel})\right], \tag{12}$$

$$I_0 = \int_0^{2\pi}\mathrm{e}^{\mathrm{i}f(\psi)}\,\mathrm{d}\psi, \qquad \mathbf{I}_{\perp} = \int_0^{2\pi}\mathbf{v}_{\perp}(\psi)\mathrm{e}^{\mathrm{i}f(\psi)}\,\mathrm{d}\psi, \tag{13}$$

$$I_{\parallel} = \int_0^{2\pi}\mathbf{v}_{\perp}{}^2(\psi)\mathrm{e}^{\mathrm{i}f(\psi)}\,\mathrm{d}\psi, \qquad \psi = \omega_0 t, \tag{14}$$

$$f(\psi) = n\psi + \omega\sqrt{\varepsilon}\Delta(\psi)/(2\omega_0) - \omega\sqrt{\varepsilon}\mathbf{n}\mathbf{x}_{\perp}(\psi), \tag{15}$$

where

$$\Delta(t) = \omega_0\int_0^t(\mathbf{v}_{\perp}^2(t') - \overline{\mathbf{v}_{\perp}^2})\,\mathrm{d}t'. \tag{16}$$

Here we use equations (10), (11) for finding the $f(\psi)$ function. The photon energy and the emission angle can be obtained from the following relations:

$$\omega = \frac{2\gamma^2 n\omega_0}{\sqrt{\varepsilon}(1+\gamma^2\theta^2+\rho/2-2\chi\gamma^2/\sqrt{\varepsilon})}, \tag{17}$$

$$\theta^2 = \frac{1}{\gamma^2}\left(\frac{2\gamma^2 n\omega_0}{\sqrt{\varepsilon}\omega}+\frac{2\chi\gamma^2}{\sqrt{\varepsilon}}-1-\rho/2\right), \tag{18}$$

where $\rho = 2\gamma^2\overline{v_\perp^2}$. Obviously, these relations are not independent and we write them for convenience of further discussion.

Equations (12)–(15) describe the spectral angular distribution of the relativistic particle radiation for the quasiperiodic motion in the transparent and isotropic medium. The trajectories of the particle are represented in these equations in a general form. Equations (12)–(15) allow us to calculate the spectral (integrated over angular variables) intensity, with the help of numerical methods, for any particle motion. However, for some general enough cases the integrals in equations (13)–(15) may be taken over angular variables as, for example, in the important case of planar motion. For planar motion

$$\mathbf{nx}_\perp = \theta\cos(\varphi)\int_0^t v_\perp(t')\,\mathrm{d}t', \tag{19}$$

where φ is the azimuthal angle. After integration over θ we get

$$\mathrm{d}I_\mathrm{p} = e^2\frac{\omega\,\mathrm{d}\omega\,\mathrm{d}\varphi}{(2\pi)^3\gamma^2}\sum_{n=-\infty}^{\infty}\vartheta(\theta^2)\left(\left\{\frac{(\varepsilon-1)}{\varepsilon}\gamma^2-1\right\}I_0^2+\gamma^2(I_x^2-I_0I_\parallel)\right). \tag{20}$$

From here on, the term $\vartheta(\theta^2)$ in the sum means that the function ϑ (which was defined after equation (5)) is equal to 0 or 1 in accordance with equation (18).

One can integrate this relation over φ and obtain the following equation for the spectral intensity:

$$\begin{aligned}\frac{\mathrm{d}I}{\mathrm{d}\omega} = &-\frac{e^2\omega}{(2\pi\gamma)^2}\sum_{n=-\infty}^{\infty}\vartheta(\theta^2)\int_{-\pi}^{\pi}\mathrm{d}t_1\,\mathrm{d}t_2\\ &\times J_0\left(2\sqrt{\varepsilon}\int_{t_2}^{t_1}\mathrm{d}\psi\, g(\psi)\sqrt{\xi(n/\sqrt{\varepsilon}-\xi(1+\rho/2-2\chi\gamma^2/\sqrt{\varepsilon}))}\right)\\ &\times\left(1-\frac{(\varepsilon-1)}{\varepsilon}\gamma^2+1/2(g(t_2)-g(t_1))^2\right)\\ &\times\cos\left((n-\sqrt{\varepsilon}\xi\rho/2)(t_1-t_2)+\sqrt{\varepsilon}\xi\int_{t_2}^{t_1}g^2(\psi)\mathrm{d}\psi\right),\end{aligned} \tag{21}$$

where $g(\psi)=\gamma[v_x(\psi)-\langle v_x\rangle]$, $v_x=v_\perp$, $\langle v_x\rangle$ is the mean transverse velocity, $\xi=\omega/(2\gamma^2\omega_0)$ and $J_0(x)$ is the Bessel function. Then we can get from equation (21) the following relation, in the dipole approximation:

$$\begin{aligned}\frac{\mathrm{d}I}{\mathrm{d}\omega} = e^2\omega&\sum_{n=-\infty,\neq 0}^{\infty}\vartheta(\theta^2)|x_n|^2\{n^2-2[\varepsilon-(\varepsilon-1)\gamma^2]\\ &\times[\xi(n/\sqrt{\varepsilon}-\xi(1+\rho/2-2\chi\gamma^2/\sqrt{\varepsilon}))]\},\end{aligned} \tag{22}$$

where $x_n = (1/(2\pi))\int_{-\pi}^{\pi}x(\psi)\exp(-\mathrm{i}n\psi)\mathrm{d}\psi$ is the Fourier component of the value $x(t) = 1/\gamma\int_0^t g(\psi)\mathrm{d}\psi$ ($x(t)/\omega_0$ is the transverse coordinate). This equation was obtained for the two conditions $4\varepsilon\gamma^2\xi^2\theta^2\rho \ll 1$ and $\rho\sqrt{\varepsilon}\xi \ll 1$. The first condition is the requirement of smallness of the argument in the Bessel function, and the second one means that the cosine in equation (21) is approximately equal to $\cos(n(t_1 - t_2))$. In spite of the fact that $\rho \ll 1$ the value $2\chi\gamma^2$ may be large and hence the argument of the Bessel function can also be large. Because of this, the first condition is also necessary. At $\varepsilon = 1$ this equation has the form of the well-known dipole approximation (when $\rho \ll 1$).

In the case when

$$2\chi\gamma^2/\sqrt{\varepsilon} > 1 + \rho/2 \tag{23}$$

the following term ($n = 0$) should be added in equation (22):

$$\frac{\mathrm{d}I_{n=0}}{\mathrm{d}\omega}(\omega) = e^2\omega\left\{\left(\frac{\varepsilon-1}{\varepsilon} - \frac{1+\rho/2}{\gamma^2}\right) - \left(\frac{(\varepsilon-1)\gamma^2}{\varepsilon} - 1\right)\right.$$
$$\left.\times\ [2\varepsilon\xi^2(2\chi\gamma^2\sqrt{\varepsilon} - (1+\rho/2))]\overline{X^2}\right\}\vartheta\left(\frac{\varepsilon-1}{\varepsilon} - \frac{1+\rho/2}{\gamma^2}\right), \tag{24}$$

where $\overline{X^2}$ is the mean square of the function $x(t)$. It should be noted that equations (23) and (24) contain an apparent contradiction, because they predict different, although very close values for the threshold of Cherenkov radiation. We explain this difference via the accuracy (up to γ^{-2} terms) of our calculations.

The equations (21)–(24) obtained here are sufficient for the calculation of the spectral intensity of the relativistic particle for planar quasiperiodic motion of the general kind in transparent media. In these equations the knowledge of the function ε is required for every computed photon energy. In particular, the process of calculation (at fixed ω) consists in testing the relation $\vartheta(\theta^2)$ for every n (in the interval $-\infty, +\infty$). For such a test, equation (18) should be used. However, it is easy to see that for the condition $2\chi\gamma^2/\sqrt{\varepsilon} < 1 + \rho/2$ only positive numbers n are possible.

4. Examples of calculations

In this section we point out the basic peculiarities of the radiation for quasiperiodic particle motion in the medium. For the detailed description of this process knowledge of the explicit form of the dielectric function is important. The aim of our consideration is the application of the equations obtained in the previous section to the calculations of radiation processes in a transparent medium. Note that many peculiarities of similar processes were discussed in earlier papers [16–19, 24, 25, 30–32].

In the general case the relations obtained here for radiation in a medium are valid, under the condition of a small influence of this medium on the quasiperiodic particle motion. Different processes (multiple scattering, ionization energy losses and others) can modify the motion of particles and they should be investigated separately. Various examples of consideration of this problem can be found in the literature [15, 21, 23]. One can assert that, in the case of small enough values of $|\varepsilon - 1|$, the influence of the medium on the motion will be insignificant, but in every specific case such a possibility should be studied. Thus, we think that in most, if not all, of the practically important cases one has $|\varepsilon - 1| \ll 1$.

It is a well-known fact that the transparent medium is an idealized substance. We assume that a good model of the transparent medium is a medium in which $\varepsilon'' \ll |\varepsilon - 1|$, where ε'' is the imaginary part of the dielectric function.

In the general case the number of harmonics n which may be radiated lies in the range $(-\infty, \infty)$. In the vacuum $n \geqslant 1$, always. However, under the condition $2\chi\gamma^2/\sqrt{\varepsilon} < 1 + \rho/2$ all the numbers obtained are positive. It is easy to see from equations (23) and (24) that the condition $2\chi\gamma^2/\sqrt{\varepsilon} = 1 + \rho/2$ is practically equal to the threshold of the Cherenkov radiation. From equation (24) it follows that, with decreasing of the amplitude of the transverse motion ($\rho \to 0$), this equation describes the intensity of the Cherenkov radiation. Besides this, $\mathrm{d}I(n=0)/\mathrm{d}\omega = 0$ at $2\chi\gamma^2/\sqrt{\varepsilon} = 1$. For $\rho \to 0$ all the remaining terms ($n \neq 0$) in equations (21) and (22) are set to zero.

Let us recall that $\rho = 2\gamma^2\overline{v_\perp^2}$ and hence we can find the threshold value of the Lorentz factor for Cherenkov radiation in the general case:

$$\gamma_{\mathrm{th}}^2 = \frac{1}{2\chi/\sqrt{\varepsilon} - \overline{v_\perp^2}}. \tag{25}$$

From here, we see that γ_{th} is increased with increase of the mean square transverse velocity. We also see that, for allowing the possibility of Cherenkov radiation, the realization of the condition $\overline{v_\perp^2} < 2\chi/\sqrt{\varepsilon}$ is necessary.

Let us consider equations (17) and (18). We see that for Cherenkov radiation ($n = 0$)

$$\theta_{\mathrm{Ch}}^2 = \frac{1}{\gamma^2}\left(\frac{2\chi\gamma^2}{\sqrt{\varepsilon}} - 1 - \rho/2\right). \tag{26}$$

This result shows that the angle of Cherenkov radiation also depends on the ρ value.

Now we consider the case of the usual amorphous media. At high enough frequencies of photons the dielectric function has the following simple form:

$$\varepsilon = 1 - \frac{\Omega_{\mathrm{p}}^2}{\omega^2} \tag{27}$$

where $\Omega_{\mathrm{p}}^2 = 4\pi n_{\mathrm{e}}e^2/m_{\mathrm{e}}$, n_{e} is the electron density and m_{e} is the electron mass. Substituting this relation in equation (27) we obtain approximately (under the condition $\Omega_{\mathrm{p}}/\omega \ll 1$) for radiation of the nth harmonic:

$$(1 + \rho/2)\omega^2 - 2\gamma^2 n\omega_0\omega + \gamma^2\Omega_{\mathrm{p}}^2 \leqslant 0. \tag{28}$$

This means that radiation (of the nth harmonic) is possible when $\gamma n\omega_0 > \Omega_{\mathrm{p}}\sqrt{1+\rho/2}$ and the resolved photon energies lie in the interval $\omega_- \leqslant \omega \leqslant \omega_+$, where

$$\omega_\pm = \frac{\gamma^2 n\omega_0 \pm \sqrt{\gamma^4 n^2\omega_0^2 - \Omega_{\mathrm{p}}^2\gamma^2(1+\rho/2)}}{1+\rho/2}. \tag{29}$$

The threshold value of the Lorentz factor for the harmonic number n is equal to

$$\gamma_{\mathrm{th}} = \frac{\Omega_{\mathrm{p}}}{\sqrt{n^2\omega_0^2 - \overline{v_\perp^2}\Omega_{\mathrm{p}}^2}} \approx \frac{\Omega_{\mathrm{p}}}{\omega_0\sqrt{n^2 - a^2\Omega_{\mathrm{p}}^2/2}}, \tag{30}$$

where a is the amplitude of the particle deflection. It is obvious that for radiation of the nth harmonic, $a\Omega_{\mathrm{p}} < \sqrt{2}n$. These results are in agreement with those in [16, 17] (for the first harmonic) and [31].

It is well known that in usual media the dielectric function is smaller than 1 at high enough photon energies. Thus, Cherenkov undulator radiation is possible mainly at photon energies $\lesssim$10 eV. Besides this, there exists the possibility of observing this radiation at the photoeffect absorption edges [24, 25]. In this case the energy of emitted photons has a value of $\lesssim$1 keV.

In accordance with quantum electrodynamics [26] the electromagnetic vacuum represents the medium in which the dielectric function may be larger than 1. However, for electric fields which may be obtained in laboratories ($<10^6$ G) the value $\varepsilon - 1$ is very small and the particles with Lorentz factors larger than 10^{10} can feel this value. In the paper [27] the Cherenkov radiation in silicon single crystals (i.e. an analogue of the quantum undulator) was predicted for particles with $\gamma > 10^8$. However, our considerations allow us to predict the specific radiation

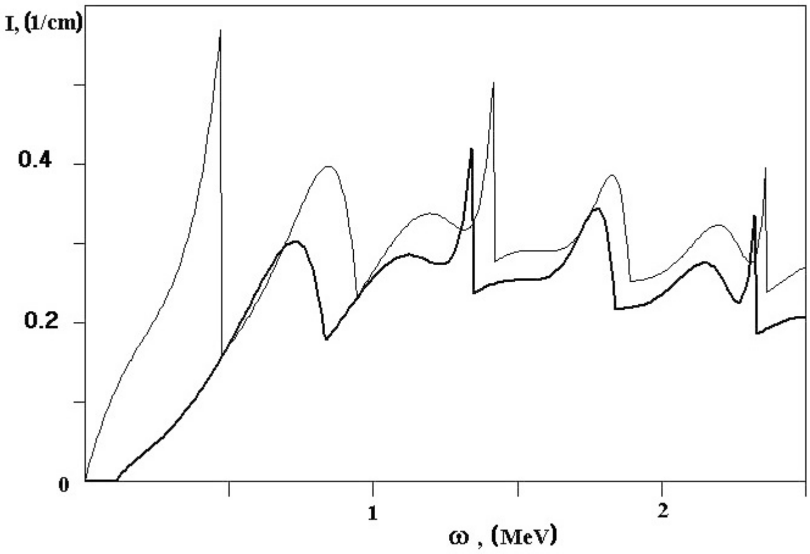

Figure 1. Intensity of radiation in a silicon crystal undulator with the period and amplitude equal to 0.05 cm and 100 Å, respectively. The energy of the positron beam is 10 GeV. Thin and thick curves correspond to radiation in vacuum and media, respectively. The parameter ρ has the value $\rho = 6.4$.

of negative harmonics in single crystals. A similar effect is also applied to the propagation of high energy charged particles in power laser waves [28, 29].

Below we present some examples of calculations of the radiation of relativistic particles for quasiperiodic motion in the medium. These calculations were done with the use of equation (21) assuming that the particle motion in the transverse plane is harmonic: $v_\perp = a\omega_0 \cos \omega_0 t$.

Figures 1 and 2 illustrate the influence of media on the radiation in the crystal undulator [12–14]. In the silicon single crystal at photon energies larger than 10 keV the dielectric function is smaller than 1. The disappearance of the first harmonic in such media is shown in figure 1. Figure 2 illustrates the influence of the medium in the case when the first harmonic is partially radiated. Notice that these figures were produced only for illustration and do not take into account many peculiarities of the real process (such as the influence of the channelling motion).

Let us consider the particle radiation in the undulator with the dielectric function larger than 1. In practice it may be a gas-filled undulator. Let the energy of particles moving in the undulator satisfy the condition for Cherenkov radiation (see equation (26)). Then the connection between the angle of radiation of the nth harmonic and the angle of the Cherenkov radiation follows from equation (18):

$$\theta^2(n) = \theta_{\mathrm{Ch}}^2 + \frac{2n\omega_0}{\sqrt{\varepsilon}\omega}. \tag{31}$$

From here, we get that the condition of radiation of the nth harmonic is

$$n > -\frac{\theta_{\mathrm{Ch}}^2 \sqrt{\varepsilon}\omega}{2\omega_0}. \tag{32}$$

Obviously all the positive n satisfy this condition and negative n satisfy equation (26) starting from some number $n_{\min}$.

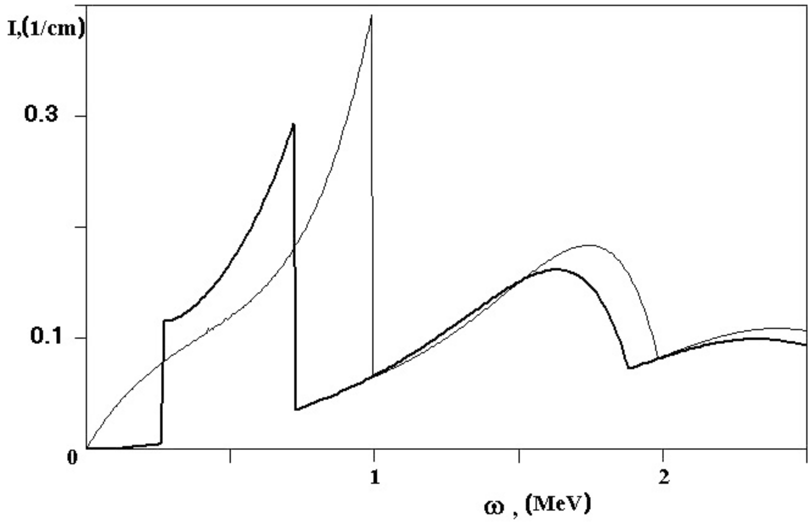

Figure 2. The same as figure 1, but with the amplitude $a = 55$ Å and $\rho = 1.83$.

Let us imagine a medium with a constant dielectric function ($\varepsilon > 1$) for all photon energies. From our consideration (see equations (25), (26), (31), (32) and the condition for the Cherenkov radiation to take place) it follows that in this medium the positive harmonic is radiated at all photon energies. There is a threshold for negative harmonics in this medium. In this case the radiation of these harmonics takes place at all above-threshold energies, and with the increase of the photon energy the number of radiated harmonics also grows. In particular, the frequency ω_0 determines only the threshold energy of radiated harmonics. This consideration shows that the character of the radiation (under the pointed conditions) is appreciably different to in the usual undulator.

For illustration of this case we carry out the calculation of the propagation of the beam with the Lorentz factor equal to 400 in the gas-filled undulator with a period equal to 10 cm. We also assume the value $\chi = 10^{-4}$ at photon energies lower than 1 eV, and $\chi = 0$ at energies higher than 1 eV. This value is several times smaller than in many gases at atmospheric pressure. The energy range of the photons corresponds approximately to visible light. Figure 3 illustrates the spectral intensity of the radiation at $\rho = 0.39$. In this case harmonics with the numbers -1, 0, 1 are predominantly radiated. We see that at small energies the radiation of the zeroth harmonic dominates (in accordance with equation (24)). The total intensity grows proportionally to the photon energy and hence is equal to the intensity of Cherenkov radiation in any medium, which is characterized by the corresponding ε value.

Figure 4 illustrates the behaviour of the intensity of the radiation, depending on the ρ parameter. One can see that at $\rho \approx 62$ the intensities of all the negative and zeroth harmonics disappear. The structure in curves at large enough ρ reflects the disappearance of the negative harmonics. The peak at $\rho \approx 38$ corresponds to the harmonic with $n = -3$. At the fixed Lorentz factor one has a threshold value $\rho_{\text{th}} = 4\chi\gamma^2/\sqrt{\varepsilon} - 2$. Figure 5 shows the angle of radiation of harmonics and the intensity of radiation at the fixed photon energy and ρ parameter. From our consideration it follows, firstly, that the undulator distributes the Cherenkov radiation over its harmonics and, secondly, that the intensity of radiation in such a medium is much higher than that in the vacuum.

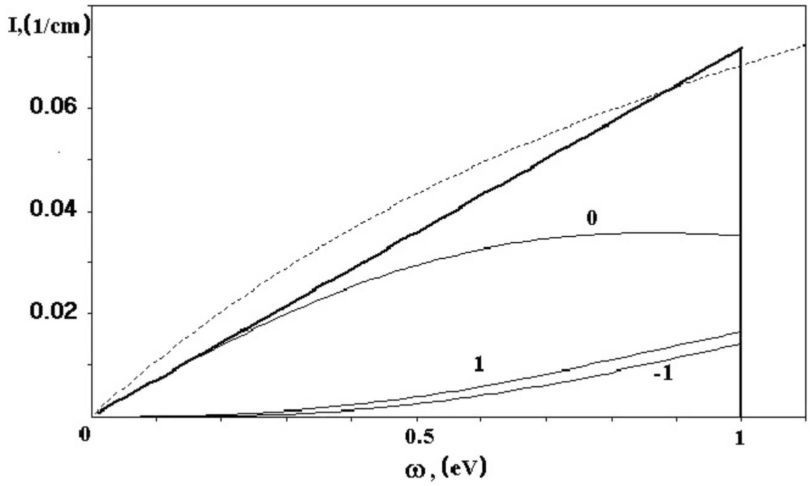

Figure 3. Intensity of radiation in a gas-filled undulator as a function of the photon energy. Curves −1, 0, 1 correspond to radiated harmonics with $n = -1, 0, 1$. The thick curve is the total intensity. The dotted curve is the intensity in vacuum enlarged 500 times (with the values of the other parameters remaining unchanged).

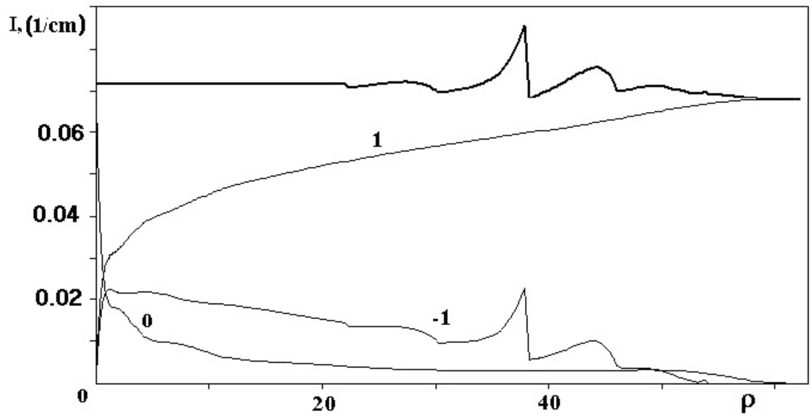

Figure 4. Intensity of radiation in a gas-filled undulator as a function of the ρ parameter. Curve 0 corresponds to the zeroth harmonic, curve 1 (−1) corresponds to the sum of intensities of all the positive (negative) harmonics. The thick curve is the total intensity. The energy of radiated photons is equal to 1 eV.

Our results for radiation in media with $\varepsilon < 1$ are in agreement with the main conclusions of the papers [16, 17].

The Cherenkov radiation for quasiperiodic motion was studied in [19]. In this paper the radiation process was considered for specific motion and different undulator parameters. However, the particular calculations and illustrations for the case $\varepsilon > 1$ and $\rho > 1$ are absent. The Cherenkov radiation was investigated in more detail at small undulator parameters. The conclusion in this paper, i.e. that the undulator radiation is negligible in comparison with the Cherenkov one, is in agreement with our results at $\rho < 1$.

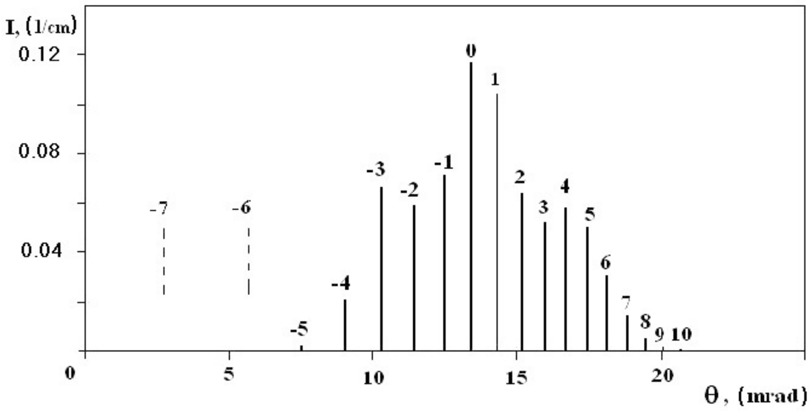

Figure 5. Radiation of the harmonics in the gas-filled undulator. The angle θ is along the abscissa axis and the intensity of radiation of the nth harmonic is along the ordinate axis. The numbers above intercepts, showing the intensity, are the numbers of the harmonics. The intensities of the sixth and seventh harmonics are invisible (due to their small values). The energy of radiated photons is equal to 1 eV. The angle θ for the zeroth harmonic is independent of the photon energy and for other harmonics these angles are changed in accordance with equation (31). The parameter ρ takes the value $\rho = 3.8$.

5. Conclusions

In this paper we considered the radiation process in a transparent medium. We obtained (on the basis of such a relation for vacuum [15]) the general relation for radiation energy losses of the relativistic particle. With the help of this formula we extended the theory [15] of the radiation for quasiperiodic motion to the case of a transparent medium. We obtained the relations describing the spectral intensity for the case of planar motion, which may be prescribed by any analytical equation. The various possibilities for radiation in transparent media were discussed.

Acknowledgments

We would like to thank R O Avakian for drawing to our attention the problem of the medium for radiation in crystal undulators. We are grateful of V G Baryshevsky and V V Tikhomirov for useful explanations concerning the first papers devoted to crystal undulators.

This work was partially supported by the Russian Foundation for Basic Research (grant 05-02-17622 and grant 05-02-08085ofi-e).

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